Quantum Computing in the Solid State.

Quantum computers can be much more efficient than conventional ones at solving certain classes of problems. Some of these problems, such as the factorisation of large numbers by Shor's algorithm have commercial applications, whilst others, like the efficient simulation of quantum systems would be a boon to research. This has led to widespread interest in how such quantum computers can be implemented. This review will discuss the requirements such a computer must have to be useful, and how these are met by the many different architectures proposed, in addition to the progress made in actually producing working computers with these architectures. We will concentrate on solid state implementations, because these architectures are the most easily scalable of the proposed architectures and they will have the benefit of fabrication techniques mastered for classical computers.

I. INTRODUCTION

Unlike a conventional computer, a quantum computer employs quantum bits, or qubits, to store information, and uses quantum properties such as interference and entanglement to aid computation. A qubit can be any two-level system, with one level designated $|0\rangle$ and the other $|1\rangle$, in deference to the classical bit. But, whereas classical bits can only be in a single definite state (i.e. only a single number can be stored at a memory location at a time), in a quantum computer, qubits can be in a superposition of many states. Each operation on a quantum computer then acts on all the states in the superposition at once, which results in a feature called *quantum parallelism*.

However quantum mechanics limits the kinds of measurements that can be made on this superposed final state. This means that not all the information from the quantum calculation can be extracted like from an equivalent classical one, because the "results" are the complex probability amplitudes of the final state. Still, the results of the calculations can be made to *interfere* with one another and hence some *global* information about the results can be obtained. This and other features, allows quantum algorithms for certain problems to be more efficient (take less time) than equivalent classical algorithms [1–3].

A practical quantum computer, however, faces many problems. The most important of this is *decoherence*, where the interaction of a quantum system with its environment suppresses interference effects in the quantum system. Thus for quantum computation a long coherence time, over which interference can occur, is desirable. Certainly this time must be much longer than the time to manipulate the qubits to perform calculations. This requirement can be quantified by a quality factor Q, defined as the ratio of the coherence time to the manipulation time.

Nonetheless, the effects of decoherence can be overcome by *fault-tolerant quantum computation*. The *threshold theorem* of this theory states that provided the noise in individual computations is below a certain threshold, it is possible to perform arbitarily large quantum computations. This threshold corresponds to a quality factor of above 10^4 [40]. Thus, provided this condition is met a quantum computation need not take less than the coherence time.

Another practical problem lies in the scalability of the quantum computer. A useful quantum computer would need to operate with hundreds, if not thousands of qubits. And scalability means that each qubit must be able to interact, or couple to each other. However, this interaction increases decoherence, and vice versa. Nonetheless solid state implementations show the most promise in this respect, because they scale easily, yet their coherence time can be made long enough. Solid state approaches will also benefit from the fabrication techniques of conventional computers.

II. SOLID STATE IMPLEMENTATION OVERVIEW

At present, two broad classes of materials are used in implementing a solid-state quantum computer: superconductors and semiconductors. We shall discuss the mechanics of these implementations, and their progress to date.

A. Superconductor implementations

Superconductor implementations [4] rely on the properties of the Josephson junction, which is a non-superconducting barrier or constriction between two superconducting elements, through which Cooper pairs, the superconducting charge carriers, can tunnel. The Josephson junction can be described by two commuting operators $\widehat{Q}$ and $\widehat{\delta}$, which represent the charge and superconducting phase difference of Cooper pairs across the junction [41]. When the area of the Josephson junction is large ($\approx 10-100\mu m^2$), the phase is well defined, so by the uncertainty relation, the charge is uncertain. In this case the junction can be described by a cubic potential.

This potential forms the basis of the *phase* qubit, whose $|0\rangle$ and $|1\rangle$ state corresponds to the ground and first excited state of this potential. The qubit is manipulated by pulses of bias current modulated at ω_{10}, corresponding to the frequency difference between the qubit's energy level. Different manipulations can be achieved by varying the duration of a pulse, analogous to the Bloch sphere construction [5]. Readout is by a differently modulated bias current pulse to excite the qubit from its $|1\rangle$ to its $|2\rangle$ state. Cooper pairs trapped in the cubic potential can then tunnel through the barrier into a "running" state, creating a measureable current. Martinis et al. have reported a fidelity of 85% for measuring the occupancy of the $|1\rangle$ from this method [6].

This qubit was first proposed by Martinis et al., who reported a coherence time of $10ns$ for a single qubit undergoing Rabi oscillations [6]. At the same time Han et al. also reported a coherence time of $4.9\mu s$ from a similar setup [7]. This discrepancy was attributed by the former to their fabrication of their qubit which introduced imperfections that drastically reduced the coherence time. Nonetheless, a similar system studied by Berkley et al. gave a coherence time of $\approx 2ns$ for both a single qubit and two capactitatively coupled qubits [8]. With a pulse frequency of the order of $10GHz$, this gives a quality factor of ≈ 10. Theory however, predicts a much higher coherence time in line with the results of Han et al., but recent work by Simmonds et al. have revealed another mechanism for decoherence which may be responsible [9].

With a smaller junction size $\approx 0.01-1\mu m^2$, the uncertainty in the phase difference and charge are about the same. This situation is used by two alternative qubit designs. The first uses an external flux generated by a control line over a loop interupted by Josephson junctions to control the qubit, and is termed the *flux* qubit. The second uses modulated pulses of bias voltage to change the charge state of the a Cooper pair box trapped between two Josephson junctions, and is called the *charge* qubit.

The flux qubit is designed so that its potential forms a double well. The states of the qubit are the two states at the bottom of each well, and correspond to classically clockwise and anti-clockwise flowing currents around the superconducting loop. The two states can interfere with each other by tunnelling through the barrier between the wells. An improved design has a superconducting quantum interference device (SQUID) as part of the loop to control the height of the barrier and hence the interference. A modulated magnetic field applied via a control line above the superconducting loop controls the qubit, and readout is by a SQUID which measures the flux of the system. This qubit design was first proposed by Bocko et al. [10] with a single junction interupting the loop. An improved design was suggested by Orlando et al. [11], with two junctions and a SQUID in the loop. Van der Wal et al. first observed the quantum interference between the two states tunnelling through the barrier in the double well [12], and more recently, Chiorescu et al. have demonstrated Rabi oscillations with a coherence time of $20ns$ with the second design. With a manipulation pulse of $3.4GHz$, this gives a quality factor of 68 [13]. Orlando et al.'s original work included numerical simulations that suggests that coherence time can exceed $0.1ms$, giving a possible quality factor of ≈ 10000.

Finally, we have the Josephson charge qubit. This qubit is formed from a Cooper pair box which is a superconducting island connected to a superconducting reservoir by Josephson junctions, and to a voltage source by a capacitor. Because the Cooper pairs in the superconducting state form a single macroscopic ground, if the number of Cooper pairs on the island change (i.e. if the charge changes), then the state of all the remaining Cooper pairs also change [14]. The two states used in this qubit are then charge states that differ by a single Cooper pair. The charge on the island can be change by apply a voltage to the capacitor to which the island is connected.

Hence, the qubit can be manipulated with modulated voltage pulses to the capacitor. Nakamura et al., and Vion et al. demonstrated this coherent manipulation by observing Rabi oscillations in the qubit, with coherence times of $2ns$ and $0.5\mu s$ respectively [15, 16]. Later experiments by Pashkin et al. demonstrated two qubit entanglement [17], followed by Yamamoto et al. who demonstrated a controlled-NOT (CNOT) gate [18]. Both of these two qubit system have coherence times of $\approx 0.6ns$, with a gate manipulation time of $\approx 100ps$, the quality factor is thus ≈ 10. However, theory suggests that the coherence time can be as high as $1\mu s$ [19], with the main source of decoherence being spontaneous photon emmision to the environment.

In practice though, the main source of decoherence is the readout mechanism which in all the experiments referenced above involved an electrode connected to the island by a Josephson junction. The excited state with one excess Cooper pair returns to ground by tunnelling through this junction so that a current is detected by the probe electrode. Whilst Nakamura, Yamamoto and Makhlin et al. used a small junction for the probe, Vion et al. used a large junction. The large junction may have caused less decoherence because it would not have coupled as much to the charge degree of freedom of the island, since the phase dominates in large junctions. Recently Astafiev et al. have demonstrated a different readout mechanism that may reduce decoherence, and also allow the single shot measurement of the qubit state, rather than an average over all the states of the qubit [20].

B. Semiconductor Qubits

Semiconductor implementations are split between those using GaAs quantum dots, or those using silicon. In both cases the qubit is either based around the spin of some particle in the system, or the charge of excess electrons in the system, similar to the Josephson charge qubit.

A quantum dot can be thought of as an artificial atom, which can contain a number of electrons bound to it. Like atoms these electrons can occupy the ground or excited states. One of the first proposals for solid state quantum computers envisaged using the ground and first excited state of a single-electron quantum dot as a qubit [21]. However, this system was found to have a theoretical coherence time of order $1ps$. This meant that normal methods of qubit manipulations, such as a modulated external electric field, could not be used because they require a manipulation time *longer* than the coherence time. However, the electric field generated by a short laser pulse could be used, since there exists lasers which can operate at sub-picosecond time scales [22].

Subsequently, many proposals were put forward to take advantage of this optical manipulation [23–25], all of which involved using an electron-hole pair, called an *exciton*, rather than just

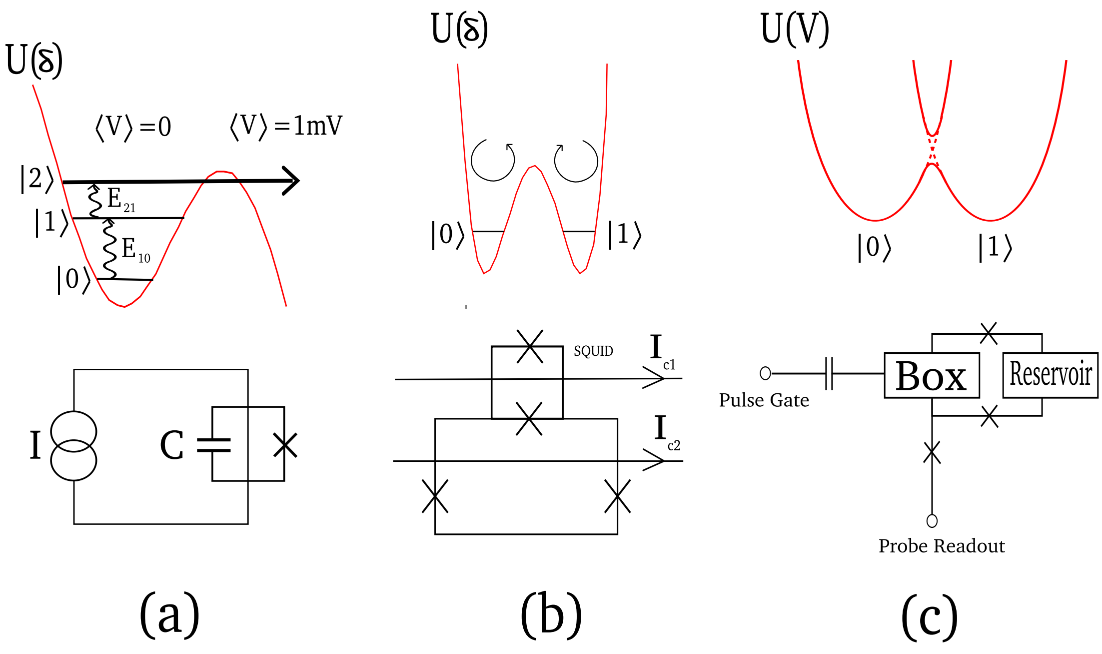

FIG. 1: *(a) Circuit diagram and potential for the phase qubit. The qubit is the lowest two states of a cubic potential well. The arrow indicates tunnelling out of the well from a state $|2\rangle$ to which qubits in state $|1\rangle$ are excited prior to readout. The tunnelling creates a measurable current outside the well. (b) Circuit diagram and potential for the flux qubit. The qubit's states are the ground states of each well in the double well potential and corresponds to clockwise and anticlockwise circulating currents. The control line above the main loop couples to both loops, and controls the height of the barrier in the double well potential. The other control only manipulates the qubit by generating a modulated magnetic field. (c) Circuit diagram and potential for the charge qubit. The qubit states corresponds to the number of excess Cooper pairs in the box. The potential as a function of applied voltage (via the pulse gate) is a set of parabolas displced by the number Cooper pairs. The Josephson energy lifts the degeneracy of the potentials (dotted line).*

the electron state of a quantum dot. An exciton is created by a laser pulse exciting a valence band electron into the conduction, leaving behind a hole. The two are then bound by the electrostatic attraction between them. The absence or presence of an exciton corresponds to the $|0\rangle$ or $|1\rangle$ states of the qubit. Readout is by the decay of the excitons which emits a photon that can be detected spectroscopically. Recent work has shown that the exciton qubit has a coherence time of $\approx 200ps$ [26–28]. With pulse times of order $1ps$, this gives a quality factor of order ≈ 100.

Recently, Li et al. demonstrated a controlled-π-rotation (CROT) gate in which two excitons in a single quantum dot acted as the two qubit. However, this system is not scalable since the excitonic energy level difference gets increasingly smaller the greater the number of excitons in the dot, so it becomes impossible to address the excitons individually [29]. In addition, Bianucci et al. have shown an implementation of the Deutsch-Josza algorithm using a single qubit [30].

Another approach, first proposed by DiVincenzo and Loss, uses the spin of an electron in a quantum dot as the qubit's two level system [31]. Under this proposal, qubit-qubit interaction is achieved by controlling the tunnelling barrier between different dots via electrical gates. When the barrier is high, no tunnelling occurs and the spin states of the qubits remain unchanged. When they are lowered, the spins become subjected to Heisenberg (spin-spin) coupling between the dots, and their spin states then evolve with time. The duration for which the barrier is lowered determines the final state of the qubits.

The problem with spin-based proposals though, is how the final state of the qubit can be detected, since no method yet exists to measure the spin of individual nuclei or electrons, only ensembles of them. However, recent work by Friesen et al. and Ono et al. suggests that such a readout may soon be practicable [32, 33]. The former suggests using spin depedent oscilations in a magnetic field which can be detected by a single electron transistor (SET), whilst the latter proposes a two electron quantum dot with one electron of fixed spin, so that any other electron tunnelling into and out of the dot has to have the opposite spin to be in the ground state due to Pauli exclusion.

The advantage of the spin proposal outlined above is the potentially long coherence time the system will have, because unlike charge based systems, it will be insensitive to charge or electric potential fluctuations in the environment. Recent work by Hanson et al. shows the coherence time to be $> 50\mu s$ [34]. With a manipulation time of $0.2\mu s$, this gives a quality factor of 250. However, theory suggests that the coherence time may be $> 1ms$, so there may be much room for improvement.

In contrast to the plethora of quantum dot proposals (especially charge based ones), there has only been one real proposal using silicon-based systems: that of Kane [35]. Progress on this implementation has recently been reviewed by Clark et al. [36]. They have been able to fabricate a prototype system using the charge states of an electron bound to two ^{31}P atoms in a silicon matrix. The $|0\rangle$ state corresponds to the electron being bound to one phosphorous atom, and the $|1\rangle$ state to the other. The position of the electron can be detected by a SET. The qubit is manipulated by electrodes above the phosphorous donor atoms. The charge proposal showed that qubit manipulation time can be as fast as $50ps$, and expects a coherence time of up to $1\mu s$, giving a quality factor of 2000 [37].

Meanwhile, spin-based silicon system, as envisaged orginally by Kane, have not had as much progress, with the major stumbling block being spin readout. However, recent work by Greentree et al. have identified a possible spin-to-charge conversion method, after which a SET can be used to read out the state of the qubit [38]. This method uses spin-dependent tunnelling of an electron between the qubit atom and a reference atom, whose charge is then detected by the SET. Tyryshkin et al. estimates that the coherence time of spin qubits in silicon can be as high as $60ms$ [39], so with a manipulation time of $< 10\mu s$ as originally envisaged by Kane, such a qubit can have a quality factor of 6000.

III. CONCLUSIONS

The quality factors of each of the proposals and implementations discussed are summarised in Figure 2, along side an arbitary measure of their progress. The progress measure encompasses the necessary steps to developing a useful quantum computer: coherent control of a single qubit (such as Rabi oscillations), two-qubit entanglement, and implementations of quantum gates. As shown in the figure, Josephson qubits are the most advanced at the moment. However, many decoherence effects [5, 9] are preventing them from realising their theoretical potential. Nonetheless, it has been shown that they can scale well [8, 18], so they may still be useful despite the low coherence time. In addition the charge qubit has demonstrated a controlled-NOT gate, which along with single qubit rotations is sufficient for any quantum computations.

The next step would be the construction of small arrays of coupled qubits (perhaps three to ten), and the implementation of algorithms such as Shor's. Judging by the pace of progress, the NEC group working on the charge qubit [15, 17, 18] may be no more than two or three years from this.

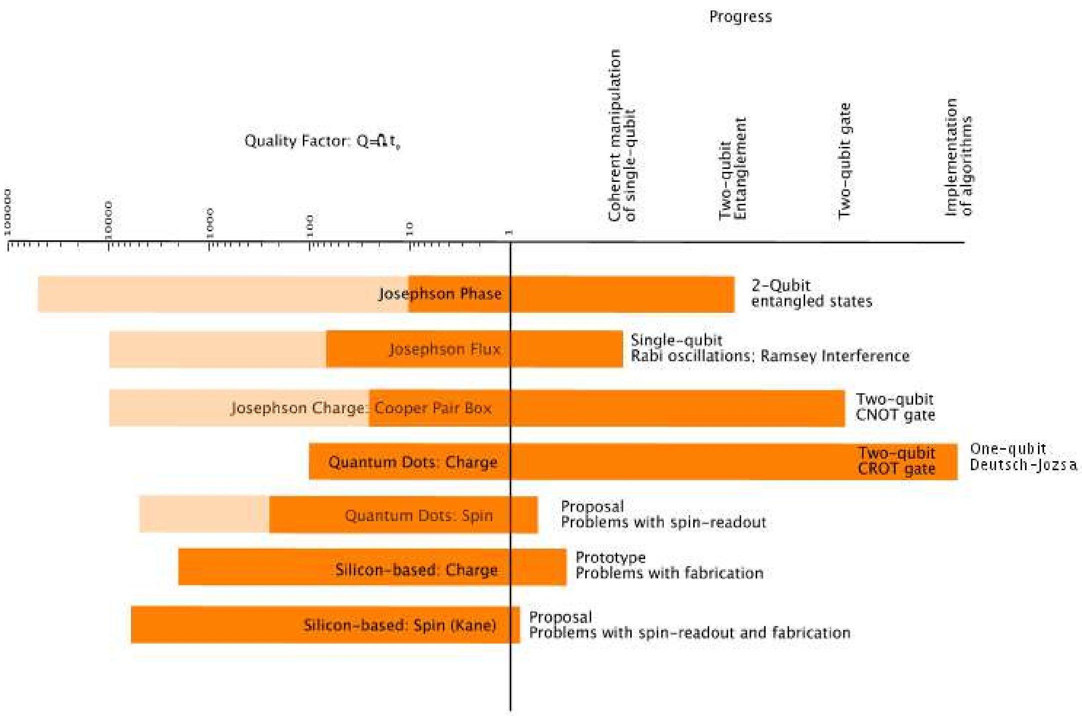

FIG. 2: *Graph of quality factor of various solid state qubit implementations (left). The light coloured bar indicates theoretical predictions of quality factor, the dark coloured bar indicates experimental data. (Right) Arbitary indication of the progress of each implementation. The CROT gate for quantum dot charge qubit was implemented by using two excitons in a single dot, and is not massively scalable.*

In addition, another group working on a similar design have reported a very high quality factor of 8000 [16] for a single qubit, that would be suitable for the implementation of fault-tolerant quantum computation. Of the alternatives, the phase qubit has yet to demonstrate a operational gate, and the flux qubit has yet to show working two qubit couplings.

Nonetheless, they have still progressed further than many semiconductor implementations apart from charge-based quantum dot designs. This later implementation though, suffers from a low coherence time, and has yet to demonstrate scalability, since the CROT gate demonstrate by Li et al. cannot scale much above three or four qubits [29]. However, for long term prospects, the spin-based semiconductor implementations may offer the greatest potential for a large scale quantum computer, since they have demonstrated very long coherence time. There are still problems with readout, and coherent control and scalability have not been demostrated. Nonetheless, Clark et al. have recently had much progress with the fabrication of large arrays of precisely placed ^{31}P donor atoms in a silicon matrix using a “bottom-up approach” [36], and several recent proposals for spin readout have been put forward [32, 33, 38]. Thus whilst they may be long in coming, spin-based quantum computers may ultimately proved more successful than other solid state alternatives.

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